# Agricultural and Resource Economics Ph.D. Qualifying Examination - Applied Microeconometrics <br> Friday, June 05, 2020 

## Instructions:

1) You will have 4 hours to complete the exam from 9 am to 1 pm .
2) There are six questions on six pages. The decision of qualifying examination committee will be based on the totality of the knowledge demonstrated over all six questions.
3) You may not use outside resources, including textbooks, notes, or electronic devices. Lockdown Browser includes a calculator feature and you should avail yourself of that tool when necessary.
4) Show all your work.
5) Leave one-inch margins and only write on one side of each sheet.
6) Do not place your name on any of your answer pages. Rather, you will be provided a "code color" that will identify your exam. Your names will not be observed by the qualifying examination committee during the grading process.
7) Clearly number each sheet with the question and page number in the upper right corner. In the upper left corner of every page write your color code.
8) Submit a scanned copy of your exam ordered by question number and page number through HuskyCT by $1: 25$ pm EDT.
9) If you need clarification about a question or believe there is a typographical error, call the committee chair at the phone number provided.
10) You may consume drinks and/or snacks, as long as doing so does not distract other students.
11) Students may use the restroom if they 1) send a text message to the committee chair in view of their camera, 2) receive an affirmative response from the chair, and 3) leave their cell phone in view of their camera.
1. Suppose that a consumer can purchase 3 goods $\left\{x_{1}, x_{2}, x_{3}\right\}$ at strictly positive prices. Let $\omega_{i}$ denote the expenditure share of good $i ; \varepsilon_{i}$ denote the expenditure elasticity of good $i$; and $\eta_{i j}$ denote the Marshallian price elasticity of good $i$ with respect to the price of good $j$. You know the following set of parameters values:

$$
\omega_{2}=\frac{2}{5} ; \varepsilon_{2}=\frac{7}{8} ; \varepsilon_{3}=\frac{9}{8} ; \eta_{23}=-\frac{5}{8} ; \eta_{32}=-\frac{3}{5}
$$

If consumer preferences satisfy a locally non-satiated rational preference ordering, Answer the following questions:
a) What share of expenditure is allocated to $x_{i}$ ?
b) What is the expenditure elasticity of $x_{1}$ ?
c) Solve for $\eta_{33}, \eta_{13}, \eta_{31}$.
2. Seven states—Arkansas, Iowa, Nebraska, North Dakota, South Dakota, Utah, and Wyoming-did not issue orders directing residents to stay at home except for essential activities in March and April 2020 in response to the coronavirus pandemic (a map of US states is provided below for geographic context). This is in contrast to the 43 other states which did issue such orders. Only one of the seven states that did not issue a stay-at-home order did not require any businesses to close: South Dakota. All seven states also closed schools to in-person instruction.


A researcher wishes to study the relationship between economic activity and state responses to Covid-19 by estimating the effect of not issuing a stay-at-home order on state-level employment. Suppose the researcher has access to state-level data on monthly employment by industrial sector, as well as demographic (e.g., age distribution, educational attainment, racial/ethnic composition, etc) and economic characteristics (e.g., share of gross state product from various industries, budget deficits, etc).
a. Propose a cross-sectional analysis. What identification problems would such an analysis face? How might the researcher respond to such problems in their analysis?
b. Propose a difference-in-difference estimator. What identification problems would such an analysis face? What analyses would you expect the researcher to conduct.
c. Propose a difference-in-difference-in-difference estimator. How would this address some of the concerns of the analyses you proposed in (a) and (b)? What analyses would you expect the researcher to conduct.
3. Define the payoff function $\omega:\{0,1\}^{3} \times\{0,1\}^{3} \rightarrow R \times R$ as $\omega\left(\sigma_{A}, \sigma_{B}\right)=\left(\omega_{A}\left(\sigma_{A}, \sigma_{B}\right), \omega_{B}\left(\sigma_{A}, \sigma_{B}\right)\right)$ where $\sigma_{i}$ denotes the strategy of player $i$ and $\omega_{i}$ denotes the payoff to player $i$ with the following values:

$$
\omega=\left[\begin{array}{ccc}
(0,2) & (4,0) & (6,3) \\
(1,4) & (2,1) & (0,1) \\
(3,0) & (0,2) & (4,0)
\end{array}\right]
$$

Does there exist a Nash equilibrium in which Player A plays a mixed strategy $\rho_{A}=\left(\rho_{1}, \rho_{2}, \rho_{3}\right)$ such that $\rho_{1} \in(0,1) ; \rho_{2}=0 ; \rho_{3} \in(0,1)$ ? If yes, provide the Nash equilibrium strategies employed by both players in such an equilibrium.
4. Let $\left\{Y, X_{1}, X_{2}\right\}$ each be column vectors of mean-zero random variables from the set of real numbers. A researcher seeks to estimate the following regression equation:

$$
Y=\beta_{1} X_{1}+\beta_{2} X_{2}+\varepsilon
$$

where $\varepsilon$ is a column vector of mean zero unobservables.
$\boldsymbol{a}$. Derive the method of moments estimators of $\beta_{1}$ and $\beta_{2}$ if $E\left[X^{\prime} \varepsilon\right]=0$, where $X=\left[\begin{array}{ll}X_{1} & X_{2}\end{array}\right]$ and the inverse of a $2 x 2$ matrix is:

$$
\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right]^{-1}=\frac{1}{a d-b c}\left[\begin{array}{cc}
d & -b \\
-c & a
\end{array}\right]
$$

Note: You must construct two closed-form equations: one for $\beta_{1}$ (denoted $\hat{\beta}_{1}$ ) and another for $\beta_{2}$ (denoted $\hat{\beta}_{2}$ ).
b. Suppose instead that $E\left[X_{1} \varepsilon\right]>0$ (but, we continue to assume that $E\left[X_{2} \varepsilon\right]>0$ ). In what direction is $\hat{\beta}_{1}$ biased in relation to $\beta_{I}$ ? In what direction is $\hat{\beta}_{2}$ biased in relation to $\beta_{2}$ ?
c. Suppose the researcher is confident that her dataset includes a variable $Z_{l}$ such that $E\left[Z_{l}{ }^{\prime} \varepsilon\right]=0$. Derive the method of moments estimator of $\beta_{l}$ if the researcher uses $Z_{1}$ as an instrument for $X_{l}$. Denote this estimate $\beta_{1, I V}$.
d. Now, suppose that $E\left[X_{2} \varepsilon\right] \neq 0$. Derive the probability limit of $\beta_{1, I V}$.
$\boldsymbol{e}$. Under what conditions is $\beta_{l, I V}$ a consistent estimate of $\beta_{l}$ ?
5. Suppose there exists an exact linear relationship between two variables $y^{*}$ and $x^{*}$ :

$$
y_{t}^{*}=\alpha+\beta x_{t}^{*}, t=1, \ldots, T
$$

But that $y^{*}$ and $x^{*}$ are measured with error, $u_{t}$ and $v_{t}$, respectively,

$$
\begin{aligned}
& \text { measured } y_{t}=y_{t}^{*}+u_{t} \\
& \text { measured } x_{t}=x_{t}^{*}+v_{t}
\end{aligned}
$$

Such that, $u_{t}$ and $v_{t}$ are iid and $u_{t} \sim N\left(0, \sigma_{u}^{2}\right)$ independently of $v_{t} \sim N\left(0, \sigma_{v}^{2}\right)$ and both measurement errors are independent of the "true" values of the variables, $y^{*}$ and $x^{*}$, being measured.
a. Show that the OLS regression of $y_{t}$ on $x_{t}$,

$$
y_{t}=\alpha+\beta x_{t}+\epsilon_{t}
$$

yield estimates of $\alpha$ and $\beta, a$ and $b$, respectively, which are inconsistent.
b. What is the direction of the inconsistency in each case, i.e., evaluate

$$
\begin{aligned}
& \operatorname{plim}(a-\alpha) \\
& \operatorname{plim}(b-\beta)
\end{aligned}
$$

(you can assume population moments and population parameters are $>0$, but an excellent answer might drop such a restriction)
c. Show that the OLS estimate of $\frac{1}{\beta}$ in the "reverse" relationship

$$
x_{t}=-\frac{\alpha}{\beta}+\frac{1}{\beta} y_{t}-\frac{\epsilon_{t}}{\beta}
$$

call this $\frac{1}{b}$ or $\tilde{b}$, is such that

$$
\operatorname{plim} b<\beta<\operatorname{plim} \tilde{b}
$$

i.e., the reciprocal of the slope from the "reverse" OLS regression and the slope from the original OLS regression "bracket" the true slope.
6. An agent has the following utility function:

$$
U\left(X_{1}, X_{2}\right)=100 *\left[\left(-A / X_{1}\right)+\left(-B / X_{2}\right)\right],
$$

where $A>0$ and $B>0$ are utility parameters. $X_{1}$ and $X_{2}$ denote the quantities of good 1 and good 2 , respectively (and asterisk [*] denotes multiplication). The agent has income of $M$ and the market price of each unit of good $i$ is $\mathrm{P}_{\mathrm{i}}$.

## Answer the following questions, supporting your argument with graphs, math, or both. For all numerical calculations, show your equations for how you obtained your answer.

(a) Given the utility function above, provide the equations that you would need to determine whether one set of prices $\left(\mathrm{P}_{1}{ }^{0}, \mathrm{P}_{2}{ }^{0}\right)$ or another set of prices $\left(\mathrm{P}_{1}{ }^{1}, \mathrm{P}_{2}{ }^{1}\right)$ would be preferred by this agent. Link these equations to a graphical example in general (graphs do not need to match this functional form precisely).
(b) Assume (for now) $A=B=1$ and prices are initially $\mathrm{P}_{1}{ }^{0}=5$ and $\mathrm{P}_{2}{ }^{0}=8$ per unit. Now, suppose there is a change in both markets such that $P_{1}{ }^{1}=7$ and $P_{2}{ }^{1}=6$ per unit. Which set of prices does this agent prefer. Explain whether your answer changes if the person's income is set to 1000 rather than some arbitrary level.
(c) Given the utility function above, provide the equation to calculate the agent's compensating variation (i.e., willingness to pay) to obtain a new set of prices ( $\mathrm{P}_{1}{ }^{1}, \mathrm{P}_{2}{ }^{1}$ ) rather than some initial set of prices ( $\mathrm{P}_{1}{ }^{0}, \mathrm{P}_{2}{ }^{\circ}$ ).
(d) Calculate the person's compensating variation for $\left(\mathrm{P}_{1}{ }^{1}, \mathrm{P}_{2}{ }^{1}\right)=(7,6)$ rather than the baseline of $\left(\mathrm{P}_{1}{ }^{0}\right.$, $\left.P_{2}{ }^{0}\right)=(5,8)$.

Suppose that utility from good 2 depends upon the level of government provision of a public good, $q$. Specifically, assume that $B=b^{*} q$.
(e) Again, assume that prices are initially $\mathrm{P}_{1}{ }^{0}=5$ and $\mathrm{P}_{2}{ }^{0}=8$ per unit. Suppose the government proposes to change its provision of $q$, so that $B$ will fall from a value of 1 to a value of 0.5 (note that the negative sign before $B$ makes this a utility-improving change, ceteris paribus). However, doing so will increase the price of good 1 from 5 to 7 while the price of good 2 remains at 8 . Calculate the agent's compensating variation for this policy change compared to the initial level of the public good (1) and ( $\left.\mathrm{P}_{1}{ }^{0}, \mathrm{P}_{2}{ }^{0}\right)=(5,8)$.
(f) Suppose instead that the government proposes to reduce the level of the public good so that B $=0.75$. Would this agent prefer the original prices $\left(\mathrm{P}_{1}{ }^{0}, \mathrm{P}_{2}{ }^{0}\right)=(5,8)$ and level of the public good (i.e., $B=1$ ), or the new prices $\left(P_{1}{ }^{0}, P_{2}{ }^{0}\right)=(7,8)$ and level of the public good (i.e., $\left.B=0.75\right)$ ? Why or why not?
(g) Using graphics of indifference curves, explain why compensating variation may not always rank two alternative sets of prices (compared to a common set of base prices) correctly, but equivalent variation always ranks price sets correctly.

